

Unit I:- DIFFERENTIAL CALCULUS

Functions, limits & differentiation & Applications

Question Bank :- of 20 Questions of 2 marks each.

Q.(1) if $f(x) = \frac{3x+1}{1-x}$; find $\frac{f(2)}{f(3)}$

Solution:- $f(x) = \frac{3x+1}{1-x}$

$$\Rightarrow f(2) = \frac{3(2)+1}{1-2} = \frac{7}{-1} = -7$$

$$\& f(3) = \frac{3(3)+1}{1-3} = \frac{9+1}{-2} = \frac{10}{-2} = -5$$

$$\Rightarrow \frac{f(2)}{f(3)} = \frac{-7}{-5} = 7/5 \text{ Ans.}$$

Q.(2) Define function with example.

Sol:- A rule denoted by f and written as

$f: A \rightarrow B$ is said to be a function from A to B ;

if for every value of $x \in A$; $\exists$ a unique $y \in B$.

For example:- $f(x) = x^2$

Q.(3) Evaluate $\lim_{x \rightarrow 0} \frac{x^{10} + x^5 + 1}{x-1}$

Sol: $\lim_{x \rightarrow 0} \frac{x^{10} + x^5 + 1}{x-1} = \frac{0+0+1}{0-1} = \frac{1}{-1} = -1$

Q. (4) Evaluate $\lim_{x \rightarrow 2} \frac{\sqrt{x} - \sqrt{2}}{x - 2}$

Solⁿ:-
$$\lim_{x \rightarrow 2} \frac{\sqrt{x} - \sqrt{2}}{x - 2} = \lim_{x \rightarrow 2} \frac{\sqrt{x} - \sqrt{2}}{x - 2} \times \frac{\sqrt{x} + \sqrt{2}}{\sqrt{x} + \sqrt{2}}$$

(Rationalization method)
$$= \lim_{x \rightarrow 2} \frac{x - 2}{(x - 2)(\sqrt{x} + \sqrt{2})}$$

$$= \frac{1}{\sqrt{2} + \sqrt{2}} = \frac{1}{2\sqrt{2}}$$

Note:- Can be solved by factorization method as $x - 2 = (\sqrt{x} - \sqrt{2})(\sqrt{x} + \sqrt{2})$

Q. (5) Evaluate $\lim_{x \rightarrow 0} \frac{\sin 3x}{x}$

Solⁿ:-
$$\lim_{x \rightarrow 0} \frac{\sin 3x}{3x} \times 3 = 1 \times 3$$

$$= 3$$

Q. (6) Write product formula of differentiation.

Solⁿ:-
$$\frac{d}{dx} \left[\underset{\text{I}}{f(x)} \cdot \underset{\text{II}}{g(x)} \right] = f(x) \cdot \frac{d}{dx} (g(x)) + g(x) \cdot \frac{d}{dx} (f(x))$$

Q. (7) Write Quotient formula of differentiation

Solⁿ:-
$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x) \cdot \frac{d}{dx} f(x) - f(x) \frac{d}{dx} g(x)}{(g(x))^2}$$

2. Differentiate $x \sin x$ w.r.t. x . (2)

solⁿ

$$y = x \sin x$$

$$\frac{dy}{dx} = \frac{d}{dx} \left[\underset{\text{I}}{x} \underset{\text{II}}{\sin x} \right]$$

$$= x \cdot \frac{d}{dx}(\sin x) + \sin x \cdot \frac{d}{dx}(x)$$

using product rule.

$$= x \cdot \cos x + \sin x \cdot 1$$

Q. (9)

Differentiate $\sin^{-1} x$ w.r.t. x .

solⁿ

$$y = \sin^{-1} x$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

Q. (10)

Differentiate $y = x^3 + e^{3x} + a^{2x}$ w.r.t. x

solⁿ

$$y = x^3 + e^{3x} + a^{2x}$$

$$\frac{dy}{dx} = \frac{d}{dx} [x^3 + e^{3x} + a^{2x}]$$

$$= \frac{d}{dx}(x^3) + \frac{d}{dx}(e^{3x}) + \frac{d}{dx}(a^{2x})$$

$$= 3x^2 + e^{3x} \cdot \frac{d}{dx}(3x) + a^{2x} \cdot \log a \cdot \frac{d}{dx}(2x)$$

$$= 3x^2 + e^{3x} \cdot 3 \cdot 1 + a^{2x} \cdot \log a \cdot 2 \cdot 1$$

$$= 3x^2 + 3e^{3x} + 2a^{2x} \log a$$

Q. (11) if $y = \frac{2x+1}{x^2-1}$, find dy/dx .

Solⁿ

$$y = \frac{2x+1}{x^2-1}$$

$$\frac{dy}{dx} = \frac{d}{dx} \left[\frac{2x+1}{x^2-1} \right]$$

$$= \frac{(x^2-1) \frac{d}{dx}(2x+1) - (2x+1) \frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$

{ using Quotient Rule }

$$= \frac{(x^2-1) \left[\frac{d}{dx}(2x) + \frac{d}{dx}(1) \right] - (2x+1) \left[\frac{d}{dx}(x^2) - \frac{d}{dx}(1) \right]}{(x^2-1)^2}$$

$$= \frac{(x^2-1) [2+0] - (2x+1)(2x-0)}{(x^2-1)^2}$$

$$= \frac{(2x^2-2) - (4x^2+2x)}{(x^2-1)^2}$$

$$= \frac{2x^2 - 2 - 4x^2 - 2x}{(x^2-1)^2}$$

$$= \frac{-2x^2 - 2x - 2}{(x^2-1)^2} \text{ Ans.}$$

differentiate $y = 3^x + x^3 + 2^x$ w.r.t. x (3)

$$y = 3^x + x^3 + 2^x$$

$$\frac{dy}{dx} = \frac{d}{dx} [3^x + x^3 + 2^x]$$

$$= \frac{d}{dx}(3^x) + \frac{d}{dx}(x^3) + \frac{d}{dx}(2^x)$$

$$= 3^x \cdot \log 3 + 3x^2 + 2^x \cdot \log 2$$

Q. (13) Differentiate $y = \sec^2 x$ w.r.t. x

Soln:-

$$y = \sec^2 x = (\sec x)^2$$

$$\frac{dy}{dx} = \frac{d}{dx}(\sec x)^2$$

$$= 2(\sec x)^{2-1} \frac{d}{dx}(\sec x)$$

$$= 2(\sec x) \cdot \sec x \tan x$$

$$= 2 \sec^2 x \tan x$$

Q. (14) Differentiate $y = 5e^x$ w.r.t. x

Soln:-

$$y = 5e^x$$

$$\frac{dy}{dx} = \frac{d}{dx} [5e^x]$$

$$= 5 \cdot \frac{d}{dx} [e^x]$$

$$= 5 \cdot e^x$$

Q.(15) Differentiate $\sin 2x - x$ w.r.t. x

solⁿ

$$y = \sin 2x - x$$

$$\frac{dy}{dx} = \frac{d}{dx} [\sin 2x - x]$$

$$= \frac{d}{dx} (\sin 2x) - \frac{d}{dx} (x)$$

$$= \cos 2x \cdot \frac{d}{dx} (2x) - 1$$

$$= \cos 2x \times 2 \times \frac{d}{dx} (x) - 1$$

$$= \cos 2x \times 2 \times 1 - 1$$

$$= 2 \cos 2x - 1$$

Q.(16) Differentiate $y = \log x$

solⁿ: - $y = \log x$

$$\frac{dy}{dx} = \frac{1}{x}$$

Q.(17) Differentiate $\sqrt{3x+4}$ w.r.t. x

solⁿ: -

$$y = \sqrt{3x+4}$$

$$y = (3x+4)^{1/2}$$

$$\frac{dy}{dx} = \frac{d}{dx} (3x+4)^{1/2}$$

$$= \frac{1}{2} (3x+4)^{-1/2} \times \frac{d}{dx} (3x+4)$$

2 marks

$$\begin{aligned}
 &= \frac{1}{2} (3x+4)^{-1/2} \times \left(3 \times \frac{d}{dx}(x) + 0 \right) \\
 &= \frac{1}{2} (3x+4)^{-1/2} \times (3 \times 1) \\
 &= \frac{1}{2} (3x+4)^{-1/2} \times 3
 \end{aligned}$$

(4)

Q.(18) if $y = \log \cos x$, find dy/dx .

solⁿ

$$y = \log \cos x$$

$$\frac{dy}{dx} = \frac{d}{dx} [\log \cos x]$$

$$= \frac{1}{\cos x} \times \frac{d}{dx} \cos x$$

$$= \frac{1}{\cos x} \times -\sin x = -\frac{\sin x}{\cos x} = -\tan x$$

Q.(19) Differentiate $y = x^2 e^x$ w.r.t. x .

solⁿ

$$y = x^2 e^x$$

$$\frac{dy}{dx} = \frac{d}{dx} (x^2 \cdot e^x)$$

$$= x^2 \frac{d}{dx} (e^x) + e^x \cdot \frac{d}{dx} (x^2)$$

$$= x^2 \cdot e^x + e^x \times 2x$$

Q.(20) find dy/dx , if $x = at^2$, $y = 2at$

solⁿ: - $x = at^2$

$$y = 2at$$

$$\frac{dx}{dt} = a \cdot \frac{d}{dt} (t^2)$$

$$\frac{dy}{dt} = 2a \frac{d}{dt} (t)$$

$$= a \times 2t$$

$$= 2a \times 1$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2a}{2at} = \frac{1}{t}$$

Exercise for practice:-

is (2 marks)

- (i) Differentiate $a^x \cos x$ w.r.t. x
- (ii) Differentiate e^x w.r.t. x
- (iii) if $y = \tan x$, find dy/dx
- (iv) if $y = (x^2 + x + 1)^{1/2}$, find dy/dx
- (v) Differentiate $\sin^2 x$ w.r.t. x .

:- DIFFERENTIAL CALCULUS

Question Bank:- of 10 Questions of 4 marks each.

Q. (1) Evaluate $\lim_{x \rightarrow 0} \frac{a^x - 1}{b^x - 1}$

Solⁿ :- $\lim_{x \rightarrow 0} \frac{a^x - 1}{b^x - 1} \times \frac{x}{x}$

$$= \lim_{x \rightarrow 0} \frac{a^x - 1}{x} \times \frac{1}{\frac{b^x - 1}{x}}$$

$$= \frac{\log a}{\log b}$$

Q. (2) Evaluate $\lim_{x \rightarrow \infty} \frac{3x^2 + x + 5}{x - 2}$

Solⁿ $\lim_{x \rightarrow \infty} \frac{3x^2 + x + 5}{x - 2}$

$$\lim_{x \rightarrow \infty} \frac{x^2 \left[3 + \frac{x}{x^2} + \frac{5}{x^2} \right]}{x^2 \left(\frac{1}{x} - \frac{2}{x^2} \right)}$$

$$= \lim_{x \rightarrow \infty} \frac{3 + \frac{1}{x} + \frac{5}{x^2}}{\frac{1}{x} - \frac{2}{x^2}}$$

$$= \frac{3 + \frac{1}{\infty} + \frac{5}{\infty}}{\frac{1}{\infty} - \frac{2}{\infty}} = \frac{3}{0} = \infty$$

Q. (3) if $y = e^x \log \sin 2x$, find $\frac{dy}{dx}$

solⁿ: -

$$y = e^x \log \sin 2x$$

marks

$$\frac{dy}{dx} = \frac{d}{dx} \left[\underset{\text{I}}{e^x} \underset{\text{II}}{\log \sin 2x} \right]$$

$$= e^x \cdot \frac{d}{dx} \log \sin 2x + \log \sin 2x \cdot \frac{d}{dx} (e^x)$$

$$= e^x \times \frac{1}{\sin 2x} \times \frac{d}{dx} (\sin 2x) + \log \sin 2x \times e^x$$

$$= e^x \times \frac{1}{\sin 2x} \times \cos 2x \times \frac{d}{dx} (2x) + \log \sin 2x \times e^x$$

$$= \frac{e^x}{\sin 2x} \times \cos 2x \times 2 \times \frac{d}{dx} (x) + \log \sin 2x \times e^x$$

$$= e^x \cot 2x \times 2 \times 1 + \log \sin 2x \times e^x$$

$$= 2 e^x \cot 2x + \log \sin 2x \cdot e^x$$

Q. (4) Find the derivative of e^x using first principle.

solⁿ: - $y = e^x$ - (i)

$$y + \Delta y = e^{x+\Delta x} \quad \text{--- (ii)}$$

$$(ii) - (i)$$

$$y + \Delta y - y = e^{x+\Delta x} - e^x$$

$$\Delta y = e^{x+\Delta x} - e^x$$

(6)

dividing both sides by Δx .

$$\frac{\Delta y}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$$

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{e^{x+\Delta x} - e^x}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{e^x \cdot e^{\Delta x} - e^x}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} e^x \left(\frac{e^{\Delta x} - 1}{\Delta x} \right)$$

using result

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = \log e$$

$$\frac{dy}{dx} = e^x \times \log e = e^x \times 1$$

$$\frac{dy}{dx} = e^x$$

Q.(5) find the derivative of $\sin x$ using first principle.

solⁿ:- $y = \sin x$ - (i)

$$y + \Delta y = \sin(x + \Delta x)$$
 - (ii)

$$(ii) - (i)$$

$$y + \Delta y - y = \sin(x + \Delta x) - \sin x$$

$$\Delta y = \sin(x + \Delta x) - \sin x$$

using result

$$\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$$

$$\Delta y = 2 \cos\left(\frac{x + \Delta x + x}{2}\right) \sin\left(\frac{\Delta x}{2}\right)$$

$$\Delta y = 2 \cos\left(\frac{2x + \Delta x}{2}\right) \sin\frac{\Delta x}{2}$$

9 marks

$$\Delta y = 2 \cos\left(x + \frac{\Delta x}{2}\right) \sin\frac{\Delta x}{2}$$

Dividing both sides by Δx

$$\frac{\Delta y}{\Delta x} = \frac{2 \cos\left(x + \frac{\Delta x}{2}\right) \sin\frac{\Delta x}{2}}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = \frac{\cos\left(x + \frac{\Delta x}{2}\right) \sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}}$$

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} \cos\left(x + \frac{\Delta x}{2}\right) \frac{\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}}$$

$$\frac{dy}{dx} = \cos\left(x + \frac{0}{2}\right) \times 1$$

$$\frac{dy}{dx} = \cos x$$

Sub. 1) Find dy/dx if $y = t^2 + 2t + 1$ (7)
 $t = 3x + 1$

Solⁿ:-

$$y = t^2 + 2t + 1$$

$$\frac{dy}{dt} = \frac{d}{dt}(t^2 + 2t + 1)$$

$$= 2t + 2 + 0$$

$$= 2t + 2$$

$$t = 3x + 1$$

$$\frac{dt}{dx} = \frac{d}{dx}(3x + 1)$$

$$= \frac{d}{dx}(3x) + \frac{d}{dx}(1)$$

$$= 3 \times 1 + 0$$

$$= 3$$

$$dy/dx = \frac{dy/dt}{dt/dx} = \frac{2t+2}{3}$$

$$= \frac{2(3x+1) + 2}{3}$$

$$= \frac{6x + 4}{3}$$

Q. (7) Differentiate $y = (\log x)^{\sin x}$ w.r.t. x

Solⁿ:-

$$y = (\log x)^{\sin x}$$

$$\log y = \log (\log x)^{\sin x}$$

$$\log y = (\sin x) \log (\log x)$$

$$\frac{d}{dx} [\log y] = \frac{d}{dx} \left[\sin x \cdot \log(\log x) \right]$$

$$\frac{1}{y} \times \frac{dy}{dx} = \sin x \times \frac{d}{dx} \log(\log x) + \log(\log x) \times \frac{d}{dx} \sin x$$

$$\frac{1}{y} \frac{dy}{dx} = \sin x \times \frac{1}{\log x} \times \frac{d}{dx} (\log x) + \log(\log x) \cos x$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{\sin x}{\log x} \times \frac{1}{x} + \log(\log x) \cdot \cos x$$

$$\frac{dy}{dx} = y \left[\frac{\sin x}{x \log x} + \log(\log x) \cdot \cos x \right]$$

$$= (\log x)^{\sin x} \left[\frac{\sin x}{x \log x} + \log(\log x) \cdot \cos x \right]$$

Q. (8) find dy/dx if $x^3 + y^3 = a^3$

Soln:- $x^3 + y^3 = a^3$

$$\frac{d}{dx} [x^3 + y^3] = \frac{d}{dx} [a^3]$$

$$\frac{d}{dx} (x^3) + \frac{d}{dx} (y^3) = 0$$

$$3x^2 + 3y^2 \times \frac{dy}{dx} = 0$$

$$3y^2 \frac{dy}{dx} = -3x^2$$

$$\frac{dy}{dx} = \frac{-x^2}{y^2}$$

Subj

1. Qⁿ :-

find $\frac{d^2 y}{dx^2}$ if $y = \sin ax$

(8)

$$y = \sin ax$$

$$\frac{dy}{dx} = \frac{d}{dx}(\sin ax)$$

$$= \cos ax \cdot \frac{d}{dx}(ax)$$

$$= \cos ax \times a \times \frac{d}{dx}(x)$$

$$= a \cos ax \times 1$$

$$= a \cos ax$$

$$\frac{d^2 y}{dx^2} = \frac{d}{dx}[a \cos ax]$$

$$= a \frac{d}{dx}[\cos ax]$$

$$= a \times -\sin ax \times \frac{d}{dx}(ax)$$

$$= -a \sin ax \times a \times \frac{d}{dx}(x)$$

$$= -a^2 \sin ax \times 1$$

$$= -a^2 \sin ax$$

Q. (10) if $y = \sqrt{x + \sqrt{x + \sqrt{x + \dots \infty}}$

solⁿ:-

$$y = \sqrt{x + \sqrt{x + \sqrt{x + \dots \infty}}}$$

(marks)

$$y = \sqrt{x + y}$$

$$y^2 = x + y$$

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(x + y)$$

$$2y \times \frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$2y \frac{dy}{dx} - \frac{dy}{dx} = 1$$

$$\frac{dy}{dx}(2y - 1) = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{2y - 1}$$

Exercise for practice:-

(i) find derivative of $y = \cos x$ using first principle

(ii) find derivative of $y = \tan x$ using first principle

(iii) find derivative of $y = x^n$ using first principle.

(iv) find derivative of $\frac{2(x+1)}{x^2 - 2x - 3}$

(v) if $y = a \sin 2x + b \cos 2x$

show $d^2y/dx^2 + 4y = 0$

(vi) differentiate $y = x^x$

(vii) differentiate $y = e^{\tan x^2}$

(viii) find dy/dx if $x = a \cos \theta$, $y = b \sin \theta$.

(ix) differentiate $\sin^{-1}(\cos x)$ w.r.t. x

(x) find d^2y/dx^2 if $y = x^3 \log x$.

Q. I:- Differential Calculus.

(9)

Question Bank:- of 5 Questions of 10 marks each

Q. (i) find the maxima and minima.

$$y = x^3 - 18x^2 + 96x$$

solⁿ:- $y = x^3 - 18x^2 + 96x \quad \text{--- (i)}$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3 - 18x^2 + 96x)$$

$$\frac{dy}{dx} = 3x^2 - 36x + 96$$

Put $\frac{dy}{dx} = 0$

$$\Rightarrow 3x^2 - 36x + 96 = 0$$

$$\Rightarrow 3(x^2 - 12x + 32) = 0$$

$$\Rightarrow x^2 - 12x + 32 = 0$$

$$\Rightarrow x^2 - 4x - 8x + 32 = 0$$

$$\Rightarrow x(x - 4) - 8(x - 4) = 0$$

$$\Rightarrow (x - 4)(x - 8) = 0$$

$$\Rightarrow x = 4, 8$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}[3x^2 - 36x + 96]$$

$$= 6x - 36$$

At $x = 4$

$$\frac{d^2y}{dx^2} \bigg|_{x=4} = 6 \times 4 - 36 = 24 - 36 = -12 < 0$$

$\Rightarrow$ function is maximum at $x = 4$

Maximum value of function

$$\begin{aligned} &= y = (4)^3 - 18(4)^2 + 96(4) \quad \left\{ \begin{array}{l} \text{Put the value} \\ x=4 \text{ in eqn (i)} \end{array} \right. \\ &= 64 - 18 \times 16 + 384 \\ &= 64 - 288 + 384 \\ &= 64 + 96 = 160 \end{aligned}$$

At $x=8$

$$\frac{d^2y}{dx^2} \bigg|_{x=8} = 6(8) - 36 = 48 - 36 = 12 > 0$$

+ve

$\Rightarrow$ function is minimum at $x=8$

$$\text{Minimum value} = y = (8)^3 - 18(8)^2 + 96(8)$$

$$y = 512 - 18 \times 64 + 768$$

$$y = 512 - 1152 + 768$$

$$y = 128$$

{ By putting value $x=8$
in eqn (i) }

Q. (2) Amongst all pairs of +ve numbers with product 64, find those whose sum is the least.

Solⁿ:- let two numbers be x and y .

$$\text{then } x \cdot y = 64$$

$$\text{let } S = x + y$$

$$\Rightarrow S = x + \frac{64}{x}$$

1) Find dy/dx , $x^y = y^x$

(10)

Solⁿ:-

$$x^y = y^x$$

$$\log x^y = \log y^x$$

$$\underbrace{y \log x}_{\text{Product Rule}} = \underbrace{x \log y}_{\text{Product Rule}}$$

$$y \times \frac{1}{x} + \log x \times \frac{dy}{dx} = x \times \frac{1}{y} \times \frac{dy}{dx} + \log y \times 1$$

$$\frac{y}{x} + \log x \times \frac{dy}{dx} = \frac{x}{y} \frac{dy}{dx} + \log y$$

$$\log x \cdot \frac{dy}{dx} - \frac{x}{y} \frac{dy}{dx} = \log y - \frac{y}{x}$$

$$\frac{dy}{dx} \left(\log x - \frac{x}{y} \right) = \log y - \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{\log y - y/x}{\log x - x/y}$$

Q. (4) if $y = x^x x^x \dots \infty$, then prove $\frac{dy}{dx} = \frac{y^2}{x(1-y \log x)}$

Solⁿ

$$y = x^y$$

$$\log y = \log x^y$$

$$\log y = y \log x$$

$$\frac{d}{dx} [\log y] = \frac{d}{dx} [y \log x]$$

$$\frac{1}{y} \times \frac{dy}{dx} = y \times \frac{1}{x} + \log x \times \frac{dy}{dx}$$

$$\frac{1}{y} \times \frac{dy}{dx} = \frac{y}{x} + \log x \cdot \frac{dy}{dx}$$

To check sum is least (minimum):

$$S = x + \frac{64}{x}$$

(marks)

$$\frac{ds}{dx} = 1 - \frac{64}{x^2}$$

$$\frac{ds}{dx} = 0, \quad 1 - \frac{64}{x^2} = 0$$

$$1 = \frac{64}{x^2}$$

$$x^2 = 64$$

$$x = \pm 8$$

$$\frac{ds}{dx} = 1 - \frac{64}{x^2} = 1 - 64x^{-2}$$

$$\begin{aligned}\frac{d^2s}{dx^2} &= 0 - 64x(-2)x^{-3} \\ &= 128x^{-3} = 128/x^3\end{aligned}$$

At $x = 8$

$$\frac{d^2s}{dx^2} = \frac{128}{8^3} > 0 \text{ (true)}$$

$\Rightarrow$ Sum is minimum at $x = 8$

$$x = 8$$

$$y = 64/x = 64/8 = 8$$

$$x, y = 8, 8.$$

$$\frac{dy}{dx} - \log x \cdot \frac{dy}{dx} = \frac{y}{x}$$

$$\frac{dy}{dx} \left[\frac{1}{y} - \log x \right] = \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{y \times y}{x(1-y \log x)} = \frac{y^2}{x(1-y \log x)}$$

Q.(5) find the maximum and minimum values of the function $y = x + \frac{1}{x}$

Solⁿ:- $y = x + \frac{1}{x}$

$$y = x + x^{-1}$$

$$\frac{dy}{dx} = 1 - x^{-2}$$

$$\frac{dy}{dx} = 1 - \frac{1}{x^2}$$

Put $dy/dx = 0$

$$1 - \frac{1}{x^2} = 0, \quad \frac{x^2 - 1}{x^2} = 0$$

$$\Rightarrow x^2 - 1 = 0, \quad x^2 = 1, \quad x = \pm 1$$

$$\begin{aligned} \text{Now } \frac{d^2y}{dx^2} &= \frac{d}{dx} \left[1 - \frac{1}{x^2} \right] = 0 - 1 \times (-2) x^{-3} \\ &= 2x^{-3} \\ &= \frac{2}{x^3} \end{aligned}$$

$$\text{At } x=1, \quad d^2y/dx^2 = 2/(1)^3 = 2 > 0$$

$\Rightarrow$ function is minimum at $x=1$

$$\text{Minimum Value ; } y = 1 + \frac{1}{1} = 1 + 1 = 2$$

At $x = -1$

$$\frac{d^2y}{dx^2} = \frac{2}{(-1)^3} = \frac{2}{(-1)} = -2 < 0$$

$\Rightarrow$ function is maximum at $x = -1$

Maximum value; $y = -1 + \frac{1}{(-1)}$
 $= -1 - 1 = -2.$

Exercise for practice:-

(i) Show that among rectangles of given area, the square has the least perimeter.

(ii) find maximum & minimum value of the function

$$y = x^3 - 6x^2 + 9x - 9$$

(iii) if $y = (\cos x)^{\log x} + (\sin x)^{\tan x}$,

then find dy/dx

Hint:- $y = u + v$

$$dy/dx = du/dx + dv/dx$$

$$u = (\cos x)^{\log x}$$

$$v = (\sin x)^{\tan x}$$

Unit-III

Integral Calculus

Question Bank (A)

very short answer type Questions (2 marks)

Q1. Solve $\int x(x^2+1) dx$

Sol: $\int (x^3+x) dx = \frac{x^4}{4} + \frac{x^2}{2} + C$

Q2. Solve $\int 2x e^{x^2} dx$

Sol: put $x^2 = t$
 $2x dx = dt$
 $\int e^t dt = e^t + C$

Q3. Write integral of $\frac{1}{a^2+x^2}$ w.r.t. x .

Sol: $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$

Q4. Evaluate $\int e^{4x} dx$

Sol: $\int e^{4x} dx = \frac{e^{4x}}{4} + C$

Q5. $\int \sec^2(3-4x) dx$

Sol: $\frac{\tan(3-4x)}{-4} + C$

Q6. $\int \sin(2x+3) dx$

Sol: $\int \sin(2x+3) dx = -\frac{\cos(2x+3)}{2} + C$

Q7. $\int (x^3+4x^2+5x+7) dx$

Sol: $= \frac{x^4}{4} + \frac{4x^3}{3} + \frac{5x^2}{2} + 7x + C$

Q 8. $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 dx$

Sol: $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 dx = \int (\sqrt{x})^2 + \left(\frac{1}{\sqrt{x}} \right)^2 + 2\sqrt{x} \frac{1}{\sqrt{x}} dx$
 $= \int \left(x + \frac{1}{x} + 2 \right) dx$
 $= \int \left(x + \frac{1}{x} + 2 \right) dx$
 $= \frac{x^2}{2} + \log|x| + 2x + C.$

Q 9. $\int (x^a + a^x + e^x a^x + \sin x) dx$

Sol: $\frac{x^{a+1}}{a+1} + \frac{a^x}{\log a} + \frac{(ea)^x}{\log ea} + x \sin x + C$

Q 10. $\int \frac{1}{4x^2+9} dx$

Sol: $\frac{1}{4} \int \frac{1}{x^2 + \frac{9}{4}} dx = \frac{1}{4} \int \frac{1}{(x)^2 + \left(\frac{3}{2}\right)^2} dx$
 $= \frac{1}{4} \left[\frac{1}{\frac{3}{2}} \tan^{-1} \left(\frac{x}{\frac{3}{2}} \right) \right] + C$
 $= \frac{2}{12} \tan^{-1} \left(\frac{2x}{3} \right) + C$
 $= \frac{1}{6} \tan^{-1} \left(\frac{2x}{3} \right) + C$

Q 11. $\int x \cos x dx$

Sol: $\int \underset{I}{x} \underset{II}{\cos x} dx$ ILATE

$= I \int II dx - \int \left(\frac{d}{dx} (I) \cdot \int II dx \right) dx$
 $= x \int \cos x dx - \int \left(\frac{d}{dx} (x) \cdot \int \cos x dx \right) dx$
 $= x \sin x - \int 1 \cdot \sin x dx = x \sin x + \cos x + C.$

Sub: $\int (3^x + \frac{1}{x^3}) dx$

Sol: $\int (3^x + x^{-3}) dx$

$$= \frac{3^x}{\log 3} + \frac{x^{-2}}{-2} + C = \frac{3^x}{\log 3} - \frac{1}{2x^2} + C$$

Q13. State Trapezoidal Rule to find area under the curve $y = f(x)$, $a < x < b$

Sol: $\int_a^b y dx = \frac{h}{2} [(y_1 + y_{n+1}) + 2(y_2 + y_3 + \dots + y_n)]$
 where $h = \frac{b-a}{n}$

Q14. $\int \sqrt{5+2x} dx$

Sol: $\int (5+2x)^{1/2} dx$
 $= \frac{(5+2x)^{1/2+1}}{(\frac{1}{2}+1) \cdot 2} + C$
 $= \frac{(5+2x)^{3/2}}{3} + C$

Q15. $\int_0^1 e^x dx$

Sol: $\int_0^1 e^x dx = [e^x]_0^1$
 $= e^1 - e^0 = e - 1$

Q16. $\int_0^{\pi/2} \cos^8 x dx$

Sol: $\int_0^{\pi/2} \cos^8 x dx = \frac{7 \times 5 \times 3 \times 1}{8 \times 6 \times 4 \times 2} \times \frac{\pi}{2} = \frac{35\pi}{256}$

Q17.

$$\int_0^{\pi/4} \frac{1}{\cos^2 x} dx$$

(1)

Sol:

$$\begin{aligned} \int_0^{\pi/4} \frac{1}{\cos^2 x} dx &= \int_0^{\pi/4} \sec^2 x dx \\ &= [\tan x]_0^{\pi/4} = \tan \pi/4 - \tan 0 \\ &= 1 - 0 = 1 \end{aligned}$$

Q18.

$$\int_{-4}^{-1} \frac{1}{x} dx$$

Sol:

$$\begin{aligned} \int_{-4}^{-1} \frac{1}{x} dx &= [\log |x|]_{-4}^{-1} \\ &= \log |-1| - \log |-4| \\ &= \log 1 - \log 4 \\ &= 0 - \log 4 = -\log 4 \end{aligned}$$

Q19.

$$\int_0^1 \frac{1}{1+x^2} dx$$

Sol:

$$\begin{aligned} \int_0^1 \frac{1}{1+x^2} dx &= [\tan^{-1} x]_0^1 \\ &= \tan^{-1} 1 - \tan^{-1} 0 \\ &= \tan^{-1} (\tan \pi/4) - \tan^{-1} (\tan 0) \\ &= \pi/4 - 0 \\ &= \pi/4 \end{aligned}$$

Q20.

$$\int \frac{1}{x^7} dx$$

Sol:

$$\begin{aligned} \int \frac{1}{x^7} dx &= \int x^{-7} dx = \frac{x^{-7+1}}{-7+1} + C = \frac{x^{-6}}{-6} + C \\ &= -\frac{1}{6x^6} + C \end{aligned}$$

ubje
Q 1

(B) (Short answer type Questions) (4 Marks)

$$\int \frac{1}{\sqrt{5x+3} + \sqrt{5x+2}} dx$$

Sol: $\int \frac{1}{\sqrt{5x+3} + \sqrt{5x+2}} \times \frac{\sqrt{5x+3} - \sqrt{5x+2}}{\sqrt{5x+3} - \sqrt{5x+2}} dx$

$$= \int \frac{\sqrt{5x+3} - \sqrt{5x+2}}{(\sqrt{5x+3})^2 - (\sqrt{5x+2})^2} dx$$

$$= \int \frac{\sqrt{5x+3} - \sqrt{5x+2}}{5x+3 - 5x-2} dx$$

$$= \int \frac{\sqrt{5x+3} - \sqrt{5x+2}}{1} dx$$

$$= \int (5x+3)^{1/2} - (5x+2)^{1/2} dx$$

$$\left(\because \int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{(n+1) \cdot a} \right)$$

$$= \frac{(5x+3)^{3/2}}{3/2 \cdot 5} - \frac{(5x+2)^{3/2}}{3/2 \cdot 5} + C$$

$$= \frac{2(5x+3)^{3/2}}{15} - \frac{2}{15} (5x+2)^{3/2} + C$$

Q 2. $\int \frac{1}{1+\sin x} dx$

Sol: $\int \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x} dx$

$$= \int \frac{1-\sin x}{1-\sin^2 x} dx$$

$$= \int \frac{1-\sin x}{\cos^2 x} dx = \int \frac{1}{\cos^2 x} dx - \int \frac{\sin x \cdot 1}{\cos x \cdot \cos x} dx$$
$$= \int \sec^2 x dx - \int \tan x \sec x dx$$

Q3. $\int \frac{3x-4}{3x^2-8x+5} dx$

Sol: $\int \frac{3x-4}{3x^2-8x+5} dx \quad \text{--- (1)}$

Let $3x^2-8x+5=t$

Diff. $(6x-8) dx = dt$

$2(3x-4) dx = dt$

$(3x-4) dx = \frac{dt}{2}$

(1) $\Rightarrow \frac{1}{2} \int \frac{1}{t} dt = \frac{1}{2} \log|t| + C$
 $= \frac{1}{2} \log|3x^2-8x+5| + C$

Q4. $\int \frac{1}{(1+x^2) \tan^{-1} x} dx \quad \text{--- (1)}$

Sol:

Let $\tan^{-1} x = t$

Diff w.r.t. x $\frac{1}{1+x^2} = \frac{dt}{dx}$

$\frac{1}{1+x^2} dx = dt$

(1) $\Rightarrow \int \frac{1}{t} dt = \log|t| + C$
 $= \log|\tan^{-1} x| + C$

Q5. $\int \frac{1}{9x^2+25} dx$

Sol:

$\frac{1}{9} \int \frac{1}{x^2 + \frac{25}{9}} dx$

$= \frac{1}{9} \int \frac{1}{(x)^2 + (\frac{5}{3})^2} dx$

$= \frac{1}{9} \left[\frac{1}{5/3} \tan^{-1} \left(\frac{x}{5/3} \right) \right] + C$

$= \frac{1}{15} \tan^{-1} \left(\frac{3x}{5} \right) + C$

$$\int \sin 3x \cos 2x \, dx$$

$$\frac{1}{2} \int 2 \sin 3x \cos 2x \, dx$$

$$= \frac{1}{2} \int [\sin(3x+2x) + \sin(3x-2x)] \, dx \quad \left\{ \because 2 \sin A \cos B = \sin(A+B) + \sin(A-B) \right.$$

$$= \frac{1}{2} \int [\sin 5x + \sin x] \, dx$$

$$= \frac{1}{2} \left[-\frac{\cos 5x}{5} - \cos x \right] + C$$

Q7. $\int \frac{3x+4}{(x-2)(x+4)} \, dx$

Sol: $\int \frac{3x+4}{(x-2)(x+4)} \, dx =$

Now $\frac{3x+4}{(x-2)(x+4)} = \frac{A}{(x-2)} + \frac{B}{(x+4)} \quad \text{--- (A)}$

$$\frac{(3x+4)}{(x-2)(x+4)} = \frac{A(x+4) + B(x-2)}{(x-2)(x+4)}$$

$$\boxed{3x+4 = A(x+4) + B(x-2)} \quad \text{--- (1)}$$

Put $x=2$ in (1)

$$6+4 = A(2+4) + B(2-2)$$

$$10 = 6A + 0$$

$$6A = 10 \Rightarrow A = \frac{10}{6} \Rightarrow \boxed{A = \frac{5}{3}}$$

Put $x=-4$ in (1)

$$-12+4 = A(-4+4) + B(-4-2)$$

$$-8 = 0 - 6B$$

$$-6B = -8 \Rightarrow B = \frac{8}{6} \Rightarrow \boxed{B = \frac{4}{3}}$$

Put A, B in (A) and integrate

$$\int \frac{3x+4}{(x-2)(x+4)} \, dx = \int \frac{5}{3(x-2)} \, dx + \int \frac{4}{3(x+4)} \, dx = \frac{5}{3} \log|x-2| + \frac{4}{3} \log|x+4| + C$$

Q8. $\int_0^{\pi/2} \cos^8 x \sin^6 x dx$ (1)

Sol: $\int_0^{\pi/2} \cos^8 x \sin^6 x dx = \frac{(7 \times 5 \times 3 \times 1)(5 \times 3 \times 1)}{(14 \times 12 \times 10 \times 8 \times 6 \times 4 \times 2)} \times \frac{\pi}{2}$
 $= \frac{35\pi}{28672}$

Using result $\int_0^{\pi/2} \sin^m x \cos^n x dx = \frac{[(m-1)(m-3)\dots + \text{ve factors}](n-1)}{(m+n)(m+n-2)\dots + \text{ve factors}} \times \frac{\pi}{2}$
 when m, n both are even

Q9. $\int_0^2 \sqrt{3x+5} dx$

Sol: $\int_0^2 (3x+5)^{1/2} dx = \left[\frac{(3x+5)^{3/2}}{\frac{3}{2} \cdot 3} \right]_0^2 = \frac{2}{9} \left[(11)^{3/2} - (5)^{3/2} \right]$
 $= \frac{2}{9} (11\sqrt{11} - 5\sqrt{5})$

Q10. Evaluate the area under the curve $y = (x^2+2)^2 + 2x$ between the ordinates $x=0$ & $x=2$ and the x -axis

Sol: Required Area $= \int_0^2 y dx = \int_0^2 ((x^2+2)^2 + 2x) dx$
 $= \int_0^2 (x^4 + 4x^2 + 4 + 2x) dx = \left(\frac{x^5}{5} \right)_0^2 + 4 \left(\frac{x^3}{3} \right)_0^2 + \left(\frac{2x^2}{2} \right)_0^2 + \left(\frac{x^2}{2} \right)_0^2$
 $= \frac{1}{5} (2^5 - 0) + \frac{4}{3} (2^3 - 0) + 4(2 - 0) + \frac{2}{2} (2^2 - 0)$
 $= \frac{1}{5} (32) + \frac{4}{3} (8) + 8 + 4$
 $= \frac{32}{5} + \frac{32}{3} + 12 = \frac{96 + 160 + 180}{15} = \frac{436}{15} \text{ sq. units}$

Sol: (C) long answer type Questions (10 marks)
 $\int x^2 \sin x$ by parts. method.

$$\begin{aligned}
 \int_I x^2 \sin x \, dx &= x^2 \int_{II} \sin x \, dx - \int \left(\frac{d}{dx} (x^2) \int \sin x \, dx \right) dx \\
 &= x^2 (-\cos x) - \int (2x) (-\cos x) \, dx \\
 &= -x^2 \cos x + \int_{I} 2x \cos x \, dx \\
 &= -x^2 \cos x + 2x \int \cos x \, dx - \int \left(\frac{d}{dx} (2x) \int \cos x \, dx \right) dx + c \\
 &= -x^2 \cos x + 2x \sin x - \int 2 \sin x \, dx + c \\
 &= -x^2 \cos x + 2x \sin x + 2 \cos x + c
 \end{aligned}$$

Q2. Apply Simpson's rule to find the approximate value of $\int_0^1 \frac{1}{1+x^2} \, dx$, taking four equal intervals
 Hence obtain an approximate value of π correct to three places of decimal.

Sol: Here we have to divide the interval $[0, 1]$ into four equal parts each of width $h = \frac{1-0}{4} = \frac{1}{4}$

Here $f(x) = y = \frac{1}{1+x^2}$ (Using $h = \frac{b-a}{n}$)

Let y_1, y_2, y_3, y_4, y_5 be the ordinates corresponding to $x = 0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}$.

$$y_1 = \frac{1}{1+0^2} = 1 \quad \left| \quad y_2 = \frac{1}{1+(\frac{1}{4})^2} = \frac{16}{17} = 0.94 \right|$$

$$y_3 = \frac{1}{1+(\frac{1}{2})^2} = \frac{4}{5} = 0.8 \quad \Bigg| \quad y_4 = \frac{1}{1+(\frac{3}{4})^2} = \frac{16}{25} \quad (i)$$

$$y_5 = \frac{1}{1+(1)^2} = \frac{1}{2} = 0.5$$

By Simpson's Rule, we get

$$\int_0^1 \frac{1}{1+x^2} dx = \frac{h}{3} [(y_1 + y_5) + 2y_3 + 4(y_2 + y_4)]$$

$$= \frac{1}{12} [(1.5) + 2(.8) + 4(.94 + .64)]$$

$$= \frac{1}{12} [1.5 + 1.6 + 6.32] = .7854 \quad \text{--- (1)}$$

Also $\int_0^1 \frac{1}{1+x^2} dx = (\tan^{-1} x)_0^1 = \tan^{-1} 1 - \tan^{-1} 0$

$$= \tan^{-1} (\tan \pi/4) - \tan^{-1} (\tan 0)$$

$$= \pi/4 - 0 = \pi/4 \quad \text{--- (2)}$$

from (1) & (2)

$$\frac{\pi}{4} = .7854$$

$$\pi = 4(.7854)$$

$$= 3.1416$$

Q3. Evaluate $\int_0^{\pi/4} \sin^4 4x \cos^2 2x dx$

Sol: Put $2x = t$ When $x = 0$, $t = 0$
 $2 dx = dt$ When $x = \frac{\pi}{4}$, $t = 2(\frac{\pi}{4}) = \frac{\pi}{2}$
 $dx = \frac{dt}{2}$

So $\int_0^{\pi/4} \sin^4 4x \cos^2 2x dx = \frac{1}{2} \int_0^{\pi/2} \sin^4 2t \cos^2 t dt$

Sub:

$$= \frac{1}{2} \int_0^{\pi/2} (2 \sin t \cos t)^4 \cos^2 t \, dt$$

$$= \frac{1}{2} \cdot 16 \int_0^{\pi/2} \sin^4 t \cos^4 t \cos^2 t \, dt$$

$$= 8 \int_0^{\pi/2} \sin^4 t \cos^6 t \, dt$$

$$= \frac{8 (4-1)(4-3)(6-1)(6-3)(6-5)}{(4+6)(4+6-2)(4+6-4)(4+6-6)(4+6-8)} \times \frac{\pi}{2}$$

Using Result

$$\int_0^{\pi/2} \sin^m x \cos^n x \, dx = \frac{[(m-1)(m-3) \dots +ve] [(n-1)(n-3) \dots +ve \text{ factors}]}{(m+n)(m+n-2) \dots +ve \text{ factors}} \times \frac{\pi}{2}$$

$$= \frac{8(3)(1)(5)(3)(1)}{(10)(8)(6)(4)(2)} \times \frac{\pi}{2}$$

$$= \frac{3\pi}{64}$$

(1)

Subject:- Applied Mathematics-II (2nd sem)

Unit III, IV :- Ordinary differential equation AND
STATISTICS

Question Bank:- of 20 Questions of 2 marks each.

Q. (1) Define differential equation with example.

solⁿ An equation containing dependent variable, independent variable and derivatives of dependent variable w.r.t. independent variable is called the differential equation.

for e.g. $dy/dx = \sin x$.

Q. (2) Define order of differential equation.

solⁿ:- The order of a differential equation is the order of the highest order derivative which occurs in it.

for e.g.:- order of diffⁿ eqⁿ $\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^3 + 2y = 0$
is 2.

Q. (3) Define degree of differential equation.

solⁿ:- The degree of the differential equation is the degree of the highest order derivative which occurs in the differential equation after the derivatives have been made free from radicals and fractions.

for e.g. Degree of diffⁿ eqⁿ $\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^3 + 2y = 0$
is 1.

Q. (4) Find the order and degree of diff

solⁿ $y = \frac{dy}{dx} + \frac{c}{dy/dx}$

$$y = \frac{(dy/dx)^2 + c}{dy/dx}$$

$$y \cdot \frac{dy}{dx} = (dy/dx)^2 + c$$

$\Rightarrow$ order = 1

degree = 2

Q. (5) Find order and degree of diffⁿ eqⁿ

$$\left[\left(1 + \left(\frac{dy}{dx} \right)^2 \right) \right]^{3/2} = 5 \frac{d^2y}{dx^2}$$

solⁿ

$$\left(\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2} \right)^{2/3} = 25 \left(\frac{d^2y}{dx^2} \right)^2$$

$$\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^3 = 25 \left(\frac{d^2y}{dx^2} \right)^2$$

order = 2

degree = 2

Q. (6) Find order, degree and linear/
Non linear diffⁿ eqⁿ. $dy/dx + 2 = y \frac{d^3y}{dx^3}$

solⁿ:- $dy/dx + 2 = y \frac{d^3y}{dx^3}$

order = 3

degree = 1

Non-linear diffⁿ eqⁿ.

Q. (7) Give an example of linear diffⁿ eqⁿ. (2)

Ans:- $\frac{d^3 y}{dx^3} + y = 0$

Q. (8) Form the differential equation of the family of curves $y = A e^{\sin^{-1} x} + 4$ where A is an arbitrary constant.

soln:- $y = A e^{\sin^{-1} x} + 4$ - (i)

$$\frac{dy}{dx} = \frac{d}{dx} [A e^{\sin^{-1} x} + 4]$$

$$\frac{dy}{dx} = \frac{d}{dx} (A e^{\sin^{-1} x}) + \frac{d}{dx} (4)$$

$$\frac{dy}{dx} = A \frac{d}{dx} (e^{\sin^{-1} x}) + 0$$

$$\frac{dy}{dx} = A \times e^{\sin^{-1} x} \times \frac{d}{dx} (\sin^{-1} x)$$

$$\frac{dy}{dx} = A e^{\sin^{-1} x} \times \frac{1}{\sqrt{1-x^2}}$$

$$\boxed{\frac{\sqrt{1-x^2} \, dy/dx}{e^{\sin^{-1} x}} = A}$$

$$y = \frac{\sqrt{1-x^2} \, dy/dx}{e^{\sin^{-1} x}} \times e^{\sin^{-1} x} + 4$$

$$y = \sqrt{1-x^2} \frac{dy}{dx} + 4$$

Q. (9) Form the diffⁿ eqⁿ of the family of curves represented by the equation $y = Ax + x^2$, where A is an arbitrary const.

solⁿ:- $y = Ax + x^2$ - (i)

$$\frac{dy}{dx} = \frac{d}{dx} [Ax + x^2]$$

$$\frac{dy}{dx} = \frac{d}{dx} (Ax) + \frac{d}{dx} (x^2)$$

$$\frac{dy}{dx} = A \cdot 1 + 2x$$

$$\boxed{\frac{dy}{dx} - 2x = A}$$

Put the value of A in eqⁿ (i)

$$y = \left(\frac{dy}{dx} - 2x \right) x + x^2$$

$$y = x \frac{dy}{dx} - 2x^2 + x^2$$

$$y = x \frac{dy}{dx} - x^2$$

Q. (10) Find the order, degree and linear / N-linear of the differential equation $\left(\frac{d^2s}{dt^2} \right)^2 + 3 \left(\frac{ds}{dt} \right)^3 + 4 = 0$

solⁿ:-

$$\text{order} = 2$$

$$\text{degree} = 2$$

Non-linear

Find the mean of 2, 8, 7, 5, 8, 3, 6, 7. (3)

$$\text{Mean } (\bar{x}) = \frac{2+8+7+5+8+3+6+7}{8}$$

$$\bar{x} = \frac{46}{8} = 5.75$$

Q. (12) Write down the formula of Coefficient of Rank Correlation.

solⁿ:-
$$r = 1 - \frac{6 \sum d^2}{n(n^2-1)}$$

where $d = R_1 - R_2$

n = no. of observations.

Q. (13) Find the median of the values
31, 38, 27, 28, 36, 25, 35, 40

solⁿ:- Arranging in ascending order

25, 27, 28, 31, 35, 36, 38, 40
 4th 5th

$n = 8$ (even)

Median = A.M. of $\left(\frac{n}{2}\right)^{\text{th}}$ & $\left(\frac{n}{2} + 1\right)^{\text{th}}$ value

= A.M. of 4th & 5th value

$$= \frac{31+35}{2} = \frac{66}{2} = 33$$

Q. (14) Find the mode from the following data: 110, 120, 130, 120, 110, 140, 130, 120, 140

solⁿ:- Since the value 120 occurs the maximum no. of times.

$$\therefore \text{Mode} = 120$$

Q. (15) Write down the formula of Coefficient of S.D. & Coefficient of Variation

solⁿ:- Coefficient of S.D. = $\frac{S.D.}{\bar{x}}$

$$\text{Coefficient of Variation} = \left(\frac{S.D.}{\bar{x}} \times 100 \right)$$

Q. (16) Write down the formula of Coefficient of Mean deviation about Mean & Median.

solⁿ:- Coefficient of Mean deviation (Mean)

$$= \frac{M.D. (\bar{x})}{\bar{x}}$$

Coefficient of Mean deviation (Median)

$$= \frac{M.D. (\text{Median})}{\text{Median}}$$

Q. (17) Write down the formula of Variance ⁽⁴⁾
Solⁿ:- Variance = $(S.D.)^2$

Q. (18) Write down the formula of Mean deviation about Mean for individual series.

Solⁿ. $M.D. (Mean) = \frac{\sum |x_i - \bar{x}|}{n}$

Where $\bar{x} = \text{Mean}$

$n = \text{no. of observations}$

Q. (19) Write down the formula of Mean for grouped data/continuous.

Solⁿ. $\text{Mean} (\bar{x}) = A + \frac{\sum f_i u_i}{\sum f_i} \times h$

Where $u_i = \frac{x_i - A}{h}$

$A = \text{Mid value of } x$

$h = \text{common difference}$

Q.(20) Write down the formula of
Mean for combined group.

Solⁿ:- if $\bar{x}_1$ and $\bar{x}_2$ are the A.M.
of two groups having n_1 and n_2 items
then

A.M. (Combined group) is

$$\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

Q1

Short answer type Ordinary Diff Eqs (4 Marks)

(5)

find the diff eqn. of which $y = e^x (A \cos x + B \sin x)$ is a solution, where A & B are arbitrary constants.

Sol.

$$y = e^x (A \cos x + B \sin x) \quad \text{--- (1)}$$

Diff (1) ^I w.r.t. ^{II} x

$$\frac{dy}{dx} = e^x (-A \sin x + B \cos x) + (A \cos x + B \sin x) e^x$$

$$\frac{dy}{dx} = e^x (-A \sin x + B \cos x) + y \quad \text{--- (2)}$$

Diff (2) w.r.t. x,

$$\frac{d^2y}{dx^2} = e^x (-A \cos x - B \sin x) + e^x (-A \sin x + B \cos x) + \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = -y + \frac{dy}{dx} - y + \frac{dy}{dx} \quad (\text{using (1) \& (2)})$$

$$\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 2y = 0.$$

Q2. find the diff eqn of $y = Ae^x + Be^{-x} + x^2$

Sol:

$$y = Ae^x + Be^{-x} + x^2 \quad \text{--- (1)}$$

$$\frac{dy}{dx} = Ae^x - Be^{-x} + 2x$$

$$\frac{d^2y}{dx^2} = Ae^x + Be^{-x} + 2 \quad \text{--- (2)}$$

from (1)

put in (2)

$$Ae^x + Be^{-x} = y - x^2$$

$$\frac{d^2y}{dx^2} = y - x^2 + 2 \Rightarrow \frac{d^2y}{dx^2} - y + x^2 - 2 = 0$$

Q3. find the A. m. for

x	10	11	12	13	14	15
f	2	6	8	6	2	6

Sol:

x_i	f_i	$f_i x_i$
10	2	20
11	6	66
12	8	96
13	6	78
14	2	28
15	6	90
	30	378

$$\begin{aligned}\bar{x} &= \frac{\sum f_i x_i}{\sum f_i} \\ &= \frac{378}{30} = 12.6\end{aligned}$$

Q4. find the mean of

x	4	6	9	10	15
f	5	10	10	7	8

Sol:

x_i	f_i	$f_i x_i$
4	5	20
6	10	60
9	10	90
10	7	70
15	8	120
	40	360

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{360}{40} = 9$$

Calculate Median of data

(6)

Sol:

X	5	6	7	8	9	10	11	12	13	15	18	20
f	1	5	11	14	16	13	10	70	4	1	1	1

X	f	C.f.
5	1	1
6	5	6
7	11	17
8	14	31
9	16	47
10	13	60
11	10	70
12	70	140
13	4	144
15	1	145
18	1	146
20	1	147
	<u>147</u>	

$$\frac{N}{2} = \frac{147}{2} = 73.5$$

$$(C.f. > 73.5)$$

Median class is -

we find that the Cumulative frequency just greater than

73.5, is 140 and the value of x corresponding to 140 is '12'. Therefore

median is 12

Q6. find the mode of following data

X	Size (inches)	30	32	34	36	38	40	42
f	No of Shirts	8	17	30	35	18	7	3

Sol: By inspection, 35 shirts are sold of size 36"

and this is the maximum number i.e. size 36" shirts have the maximum sale.

• Mode of this distribution = 36.

Q7. find the mode from the following data :

110, 120, 130, 120, 110, 140, 130, 120, 140, 120

Sol: Arranging the data in the form of a frequency table, we have

value :	110	120	130	140
frequency :	2	4	2	2

Since the value 120 occurs the maximum number of items. Hence the modal value is 120.

Q8. Calculate M.D. of data (about mean)

X 13, 16, 15, 15, 18, 15, 14, 18, 16, 10. Also coeff of M.D.

Sol:

X	$ X - \bar{X} $
13	2
16	1
15	0
15	0
18	3
15	0
14	1
18	3
16	1
10	5
<u>150</u>	<u>16</u>

$$\bar{X} = \frac{\sum X}{n} = \frac{150}{10} = 15$$

$$\text{M.D. (about mean)} = \frac{16}{10} = 1.6$$

$$\text{Coeff of M.D.} = \frac{\text{M.D.}}{\bar{X}} = \frac{1.6}{15}$$

for find the mean deviation about the median (+)

x	5	7	9	10	12	15
f	8	6	2	2	2	6

Sol:

X	f	Cf	x - median	f x - median
5	8	8	2	16
7	6	14	0	0
9	2	16	2	4
10	2	18	3	6
12	2	20	5	10
15	6	26	8	48
	<u>26</u>			<u>84</u>

Median = size of

$$\frac{N}{2} = \frac{26}{2} = 13$$

$$C.f. > 13 = 14$$

and the value of x corresponding to 14 is 7.

∴ so median is 7.

$$M.O.D. (\text{median}) = \frac{\sum f |x - \text{median}|}{N = \sum f}$$

$$= \frac{84}{26} = 3.23$$

Q 10.

Calculate S.D. of data

13, 16, 15, 15, 18, 15, 14, 18, 16, 10

Sol:

X	$ X - \bar{X} $	$ X - \bar{X} ^2$
13	2	4
16	1	1
15	0	0
15	0	0
18	3	9
15	0	0
14	1	1
18	3	9
16	1	1
10	5	25
<u>150</u>		<u>50</u>

$$\bar{X} = \frac{\sum X}{n} = \frac{150}{10} = 15$$

$$S.D. = \sqrt{\frac{\sum |X - \bar{X}|^2}{n}} = \sqrt{\frac{50}{10}} = \sqrt{5} = 2.23$$

1. Calculate mean deviation and its coeff from the (8) mean for the following data:

C.I.	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
freq	5	8	7	12	28	20	10	10

Sol:

C.I.	f	Mid pt of CI x	fx	$ x - \bar{x} $	$f x - \bar{x} $
0-10	5	5	25	40	200
10-20	8	15	120	30	240
20-30	7	25	175	20	140
30-40	12	35	420	10	120
40-50	28	45	1260	0	0
50-60	20	55	1100	10	200
60-70	10	65	650	20	200
70-80	10	75	750	30	300
$N = 100$		$\Sigma fx = 4500$		$\Sigma f x - \bar{x} = 1400$	

$$\therefore \bar{x} = \frac{\Sigma fx}{N} = \frac{4500}{100} = 45$$

$$M.D.(\bar{x}) = \frac{\Sigma f|x - \bar{x}|}{N} = \frac{1400}{100} = 14$$

$$\text{and Coeff of } M.D.(\bar{x}) = \frac{M.D.(\bar{x})}{\bar{x}}$$

$$= \frac{14}{45} = 0.31.$$

Find the S.D. and C.V. for the following frequency distribution.

class intervals	frequency (f)	(x)	(fx)	(x - $\bar{x}$)	(x - $\bar{x}$) ²	f(x - $\bar{x}$) ²
4-8	11	6	66	-14	196	2156
8-12	13	10	130	-10	100	1300
12-16	16	14	224	-6	36	576
16-20	14	18	252	-2	04	56
20-24	14	22	308	2	04	56
24-28	9	26	234	6	36	324
28-32	17	30	510	10	100	1700
32-36	6	34	204	14	196	1176
36-40	4	38	152	18	324	1296

$$\Sigma f = 104 \quad \Sigma fx = 2080$$

$$\Sigma f(x - \bar{x})^2 = 8640$$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{2080}{104} = 20$$

$$S.D. = \sqrt{\frac{\Sigma f(x - \bar{x})^2}{\Sigma f}} = \sqrt{\frac{8640}{104}} = \sqrt{83.08} = 9.11$$

$$C.V. = \frac{S.D.}{\bar{x}} \times 100 = \frac{9.11}{20} \times 100 = 45.55$$

Long answer type Questions (10 Marks) (10)

3. find the coeff of rank correlation for the data ;

X	56	58	62	72	54	52	71	90	81	92
Y	15	12	16	17	19	20	21	22	23	24

Sol:

X	R ₁	Y	R ₂	d = R ₁ - R ₂	d ²
56	8	15	9	-1	1
58	7	12	10	-3	9
62	6	16	8	-2	4
72	4	17	7	-3	9
54	9	19	6	3	9
52	10	20	5	5	25
71	5	21	4	1	1
90	2	22	3	-1	1
81	3	23	2	1	1
92	1	24	1	0	0

$$\Sigma d^2 = 60$$

$$r = 1 - \frac{6 \Sigma d^2}{n(n^2 - 1)}$$

$$= 1 - \frac{6(60)}{10(100 - 1)} = 1 - \frac{360}{990}$$

$$= \frac{990 - 360}{990} = \frac{630}{990} = \frac{21}{33} = 0.64$$

Important Questions

Q1 Define an Explicit fn & give one example.

Q2 What are the values of $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a}$ and $\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}}$

Q3 Evaluate $\lim_{x \rightarrow 4} \frac{x^3 - 64}{x - 4}$

Q4 Evaluate $\lim_{x \rightarrow 0} \frac{e^{3x} - 1}{x}$

Q5 Evaluate $\lim_{x \rightarrow \infty} \frac{x^2 + x + 2}{2x^2 + 3x + 7}$

Q6 Differentiate $\sin^2 x$ w.r.t. x .

Q7 If $y = (x^2 + x + 1)^{\frac{1}{2}}$ find $\frac{dy}{dx}$.

Q8 Diff $(1+x)^2$ w.r.t. x .

Q9 Diff $y = \cos x + \log x$ w.r.t. x .

Q10 Write The Product Rule and Quotient Rule of differentiation.

Q11 If $s = ut + \frac{1}{2}at^2$ and velocity at any time is given by $\frac{ds}{dt}$, find it.

Q12 If $f(x) = \frac{1 - \tan x}{1 + \tan x}$ find $f(45^\circ)$

Q13 $\lim_{x \rightarrow 0} \frac{\tan ax}{\tan bx}$

Q14 Diff $y = (x^2 + 1)(x^4 + 3)$ w.r.t. x .

Q15 If $y = \log(\sin x)$ then find $\frac{dy}{dx}$.

Q16 If $y = \frac{1 + \log x}{1 - \log x}$ find $\frac{dy}{dx}$.

Q17 $\lim_{x \rightarrow 5} \frac{\sqrt{x} - \sqrt{5}}{x - 5}$.

Q18 $\lim_{x \rightarrow 0} \frac{\tan 3x - 2x}{3x - \sin^2 x}$.

Q19 Diff $\frac{e^x}{1+x^2}$ w.r.t. x .

Q20 Diff x^x w.r.t. x .

Q21 find $\frac{d^2y}{dx^2}$ if $y = (\log x)^2$.

Q22 Diff $\cos x$ by using ab-initio method.

Q23 Diff $y = (\tan x)^{\cos x}$ w.r.t. x .

Q24 find $\frac{d^2y}{dx^2}$ if $y = e^x \sin 5x$.

Q25 Evaluate $\lim_{x \rightarrow 0} \frac{\sin 2x}{e^x - 1}$.

Q26 If $y = \sqrt{x + \sqrt{x + \sqrt{x + \dots \infty}}}$ find $\frac{dy}{dx}$.

Q27 If $y = x + \tan x$

Prove that $\cos^2 x \frac{d^2y}{dx^2} - 2y + 2x = 0$.

Q28 find slope of tangent to the curve $y = 3x^2 + 1$ at $x = 2$.

Q30

Diff $y = x^2 e^{2x}$ w.r.t. x .

find $\frac{dy}{dx}$ if $x = at^2$, $y = 2at$

Q31

find $\frac{dy}{dx}$ if $x = a \cos \theta$, $y = b \sin \theta$.

Q32

find the slope of tangent and normal to curve $x^2 + 3y + y^2 = 5$ at $(1, 1)$.

Q33

find slope to the curve $y = x - \frac{1}{x}$ at $x = 1$

Q34

If $y = \sec(\tan^{-1} x)$ then find $\frac{dy}{dx}$.

Q35

If $y = \cot x$ Prove that $\frac{d^2y}{dx^2} + 2y \frac{dy}{dx} = 0$.

Q36

Determine the points on the curve $2y = 3 - x^2$ at which tangent is parallel to line $x + y = 0$

Q37

If $y = x^{n-1} \cot x$ find $\frac{dy}{dx}$

Q38

Diff e^x by using first principle.

Q39

If $y = \tan^{-1} x$ Prove that

$$(1+x^2) \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} = 0.$$

Q40

$$\lim_{\theta \rightarrow 0} \frac{\sin m\theta}{\sin n\theta}$$

Q41

find Maxima & Minima pts & corresponding values of $y = 2x^3 - 21x^2 + 36x - 20$.

Integral Calculus

Short answer type - (2) marks.

Solve $\int x(x^2+1) dx$

Integrate $\int 2x e^{x^2} dx$

Write integral of $\frac{1}{a^2+x^2}$ w.r.t. x

Evaluate $\int_0^1 e^x dx$.

State trapezoidal Rule to find area under the curve $y = f(x)$, $a < x < b$.

$\int \sec^2(3-4x) dx$

$\int \sin(2x+3) dx$

$\int (x^3+4x^2+5x+7) dx$

$\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 dx$

$\int_0^{\pi/2} \cos^2 x dx$

$\int \sqrt{1-\sin 2x} dx$

$\int \tan^2 x dx$

$\int (x^a + a^x + e^x a^x + \sin x) dx$

$\int \frac{1 dx}{4x^2+9}$

$\int x \cos x dx$

$\int_0^{\pi/4} \frac{1}{\cos^2 x} dx$

Q17 $\int \left(3^x + \frac{1}{x^3}\right) dx$

18 $\int_{-4}^{-1} \frac{1}{x} dx$

19 $\int_0^1 \frac{1}{1+x^2} dx$

20 $\int \sqrt{5+2x} dx$

(4 mks) Questions

1 Evaluate $\int_0^{\pi/2} \cos^8 x \, dx$

2 $\int x^2 \cos x$ by parts

3 $\int \frac{1}{(1+x^2) \tan^{-1} x} \, dx$

4 $\int_0^{\pi/2} \frac{1}{1+\cos x} \, dx$

5 $\int \frac{1}{(x+1)(x+3)} \, dx$

6 $\int \frac{6x-8}{3x^2-8x+5} \, dx$

7 $\int \frac{1}{9x^2+25} \, dx$

8 $\int \sin 3x \cos 2x \, dx$

9 $\int x \sin 2x \, dx$

10 $\int \frac{3x+4}{(x-2)(x+4)} \, dx$

11 $\int_0^1 \frac{1}{1+x^2} \, dx$

12 $\int \frac{x^3 + 2x^2 + x + 3}{x^2 + 1} \, dx$

13 $\int \frac{\sec x}{\log(\sec x + \tan x)} \, dx$

14 $\int \frac{4x+1}{x^2+3x+2} \, dx$

15 $\int x^2 \cos x \, dx$

16 $\int_{-1}^1 \sin^5 x \cos^4 x \, dx$

17 $\int \log x \, dx$

18 $\int \frac{1}{\sqrt{5x+3} + \sqrt{5x+2}} \, dx$

19 $\int \frac{3x^5}{1+x^{12}} \, dx$

20 $\int \frac{2}{x^2-5x+6} \, dx$

21 $\int_0^{\pi/2} \frac{1}{1+\cos x} \, dx$

22 $\int \frac{\tan^{-1} x}{1+x^2} \, dx$

23 $\int x \sin x \, dx$

24 $\int \frac{1}{(x-7)(x-2)} \, dx$

25 $\int_0^{\pi/2} \cos^8 x \, dx$

26 $\int \frac{1}{1+\sin x} \, dx$

(long answer type) (10 marks)

Q1 Solve $\int x^2 \sin x$ by parts.

2 $\int_0^1 \frac{1}{1+x^2}$ by Simpson's Rule taking ten equal intervals.

3 Evaluate $\int e^x \cos 2x$ by parts.

4 Use trapezoidal rule to estimate $\int_0^7 x^2 dx$ by taking 8 ordinates.

5 Evaluate $\int e^{2x} \sin 5x dx$

6 Evaluate $\int_0^{\pi/4} \sin^4 4x \cos^3 2x dx$ Hint: $2\theta = \pi/4$

7 Evaluate $\int \frac{\sec^2 x}{(2+\tan x)(3+\tan x)} dx$

8 Evaluate $\int_0^6 y dx$ by Simpson's Rule when

x	0	1	2	3	4	5	6
y	0.146	0.161	0.176	0.190	0.204	0.217	0.230

9 Evaluate $\int \frac{x-1}{x^3+1} dx$

10 Same as Q4.

Ordinary Diff Eqn & statistics

2 Mks Questions

1 Define degree of diff eqn with example

2 Give an example of linear diff eqn

3 Write Order, degree, linear/non linear of diff eqn

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 5y = 0$$

4 form a diff eqn $y = A \sin x$

5 Write the formula of rank correlation coefficient.

6 find order and degree of diff eqn

$$\frac{d^2y}{dx^2} + 3y = 0$$

7 Solve the diff eqn $\frac{dy}{dx} = x$

8 Write the formula for Coeff of variation (C.V)

9 find mean of daily wages of 10 workers
13, 16, 15, 15, 18, 15, 14, 18, 16, 10.

10 Define degree and order of diff eqn.

11 Write Order and degree of diff eqn of

$$\frac{d^2y}{dx^2} = \sqrt{1 + \frac{dy}{dx}}$$

12 find the median of 4, 6, 7, 11, 18.

13 find S.D. for 12, 6, 7, 3, 15, 10, 18, 5

14 Write formula of rank correlation coeff.

15 Write order and degree of diff eqn

$$(x+1) \frac{dy}{dx} + 2y = x$$

16 Determine the solution of diff eqn

$$xy dx + yx dy = 0$$

17 Calculate the median of data

13, 14, 16, 18, 20, 22.

18 Calculate the mean of 1, 2, 3, 4, 5, 6.

19 find the mode of data

110, 120, 130, 120, 110, 140, 130, 120, 140, 120.

20 Same 14 Q

21 Median of 7, 6, 4, 11, 20, 19, 18

22 Write order and degree of

$$\frac{d^3y}{dx^3} + 2 \frac{d^2y}{dx^2} + 4 \frac{dy}{dx} = \sin x$$

23 find the diff eqn of curve $y = Ae^{2x} + Be^{-x}$
where A and B are arbitrary constant

24 find the A.M. of marks obtained by
given 10, 12, 15, 18, 8, 11, 13, 17, 19, 14.

4mks Questions

1 find the mean of

X :	4	7	10	13	16	19
f :	7	10	15	20	25	30

2 find the S.D. of 12, 6, 7, 3, 15, 10, 18, 5.

3 Show that $y = e^x + 1$ is a solution of diff eqn.

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$$

4 form the diff eq by eliminating the arbitrary constants from

$$y = Ae^{3x} + Be^{-3x}$$

5 Calculate the median from the table :

variable	2	4	6	8	10	12	14	16
frequency	5	8	13	15	16	11	6	4

6 find out the Standard Deviation (S.D.) for 5, 8, 7, 11, 9, 10, 8, 2, 4, 6, 7.

7 Wages of six persons in rupees are 780, 690, 1200, 510, 1080, 1500. find Mean deviation (M.D.)

8 find the diff eq of family of curves
 $y = ax^2 + bx + c$

9 solve the diff. eq
 $x \log x \, dy - y \, dx = 0$

10 find the A.M. of

x : 10 11 12 13 14 15

f : 2 6 8 6 2 6

11 Define median and mode with example.

12 Solve the diff eq

$$\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0.$$

13 Calculate the mode for

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
freq.	5	8	7	12	28	20	10	10

14 Obtain the mean of

x : 4 6 9 10 15

f : 5 10 10 7 8

15 find mean deviation about mean for

x_i : 10 30 50 70 90

f_i : 4 24 28 16 8

16 form Diff. eq of $y = Ae^{2x} + Be^{-2x}$.

17 Solve the diff eq

$$(1+x)y \, dx + (1-y)x \, dy = 0.$$

18 find the mean of data

x : 25 30 35 40

f : 12 10 8 20

19 find median of data

x : 0-8 8-16 16-24 24-32

f : 8 7 16 24 15 7

10 Mks Question

1. Compute S.D. of data

x	1	2	3	4	5	6	7
f	6	12	18	26	16	10	8

2. Ten students were examined in Economics and statistics the rank obtained by students are

Ecc	1	2	3	4	5	6	7	8	9	10	(4)
stati	2	4	1	5	3	9	7	10	6	8	

3. find the variance and standard deviation from the following frequency distribution

variable x_i	2	4	6	8	10	12	14	16
freq f_i	4	4	5	15	8	5	4	5

4. Compute S.D. & C.V of following data

CI	10-20	20-30	30-40	40-50	50-60	60-70
freq	6	15	12	10	6	3

5. In a poetry recitation competition 10 participants were accorded following marks by two judges X and Y

X	15	17	14	13	11	12	16	18	10	9
Y	15	12	4	6	7	9	3	10	2	5

Calculate the Coeff of rank correlation (2).

6. find Coeff of rank correlation of

Practicals	8	3	9	2	7	10	4	6	1	5
Theory	9	5	10	1	8	7	3	4	2	6